VIBRATIONAL RELAXATION DURING MIXING OF UNDESIGNED

TWO-DIMENSIONAL STREAMS

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Theoretical research of the mixing of gases taking into account vibrational relaxation is of extreme importance today, especially in relation to the study of hydrodynamic lasers (HDL) with selective thermal excitation. These lasers find application in metallurgical production, where the nitrogen necessary for their operation is available in large quantities, and they can be activated with heat exchangers [1]. For numerical study of the processes behind a HDL one usually uses a simple model for instantaneous mixing, which, however, does not reflect the real features of mixing. A model which is based on the equations of a boundary layer or a narrow channel would more accurately describe the process of mixing, but it is, then, not possible to investigate the mixing of supersonic streams with different pressures. Therefore, it is more useful to apply the simplified Navier–Stokes equation, which have recently gained popularity.

1. This study will consider laminar and turbulent mixing in a periodic system of two-dimensional supersonic streams of CO_2–H_2O and N_2 taking into account vibrational nonequilibrium and forced radiation. Numerical modeling is done using the simplified Navier–Stokes or Reynolds equations for turbulent flows

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0,\ \frac{\partial}{\partial x}(\rho u^2)+\frac{\partial}{\partial y}(\rho uv)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right),$$

$$\frac{\partial}{\partial x}(\rho uv)+\frac{\partial}{\partial y}(\rho v^2)=-\frac{\partial p}{\partial y}+\frac{4}{3}\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right),$$

$$\frac{\partial}{\partial x}(\rho uh)+\frac{\partial}{\partial y}(\rho vh)=u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial y}\left\{\frac{\mu}{\Pr}\sum_{i=1}^{3}\left[(\mathrm{Le}_i-1)h_i\frac{\partial c_i}{\partial y}+c_i\frac{\partial e_{vi}}{\partial y}\right]\right\}-\alpha J,$$

$$\frac{\partial}{\partial x}(\rho uc_i)+\frac{\partial}{\partial y}(\rho vc_i)=\frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\mathrm{Le}_i\frac{\partial c_i}{\partial y}\right),$$

$$\frac{\partial}{\partial x}(\rho uc_1(\varepsilon_1+2\varepsilon_2))+\frac{\partial}{\partial y}[\rho vc_1(\varepsilon_1+2\varepsilon_2)]=\frac{\partial}{\partial y}\left[\frac{\mu}{\Pr}\mathrm{Le}_1\frac{\partial}{\partial y}c_1(\varepsilon_1+2\varepsilon_2)\right]+$$

$$Q_1+\frac{2\alpha Jm}{(\Theta_3-\Theta_1)R},\ \frac{\partial}{\partial x}(\rho uc_1\varepsilon_3)+\frac{\partial}{\partial y}(\rho vc_1\varepsilon_3)=\frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\mathrm{Le}_1\frac{\partial c_1\varepsilon_3}{\partial y}\right)+$$

$$Q_3-\frac{\alpha Jm}{(\Theta_3-\Theta_1)R},\ \frac{\partial}{\partial x}(\rho uc_2\varepsilon_4)+\frac{\partial}{\partial y}(\rho vc_2\varepsilon_4)=\frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\mathrm{Le}_2\frac{\partial c_2\varepsilon_4}{\partial y}\right)+Q_4,$$

where α is the amplification factor, m is the molecular weight of CO_2, J is the radiation intensity, and θ_j is the characteristic temperature of the j-th vibrational mode. The remaining definitions and details of the problem are taken from [2]. The conditions for two uniform flows are assigned to the initial cross section. The conditions of symmetry apply at the boundaries of the designed region.

Calculation of the turbulent states is done using the hypothesis regarding effective viscosity [3], where one uses the equation for the kinetic energy of turbulent pulsations

$$\frac{\partial}{\partial x}(\rho ub)+\frac{\partial}{\partial y}(\rho vb)=\frac{\partial}{\partial y}\left(\mu_t\frac{\partial b}{\partial y}\right)+\mu_t\left(\frac{\partial u}{\partial y}\right)^2-w_b$$

and the Kolmogorov–Prandtl relation

$$\mu_t=c\rho l_t\sqrt{b}.$$

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The turbulent analog of the Prandtl number is assumed equal to 0.7, and the Lewis number is equal to one.

The simple model from [4, 5] is used for describing the interaction of forced radiation in the resonator with a medium. It is assumed that the intensity of the forced radiation is constant. The energy output is calculated using the equation

$$W = Jt_1 s_p/[(1 + r_1)G],$$

where t_1 is the transmission factor of a translucent mirror, r_1 is the mirror's reflection factor, s_p is the area of the mirror, and G is the flow rate of the mixture of all the gases through the resonator. The parameters for the mirror are related by the equation $t = 1 - a - r$, where the absorption coefficients are $a_1 = a_2 = 0.02$. The coefficient t_1 is determined from the condition of steady-state generation

$$\frac{1}{L_x L_y}\iint_s \alpha dx dy = \alpha^*(t_1) \equiv \frac{1}{L_y}\left(\left(\frac{1-r_1}{1+r_1}\right) + \left(\frac{1-r_2}{1+r_2}\right)\right),$$

where α^* is the threshold amplification factor, L_x is the dimension of the resonator "along the flow," and L_y is the dimension of the resonator "along the beam." It is assumed that the streams of $CO_2 + H_2O$ and N_2 alternate in the direction "along the beam," and, therefore, for determining the average amplification factor integration is done over the cross section of the resonator in a direction perpendicular to the planes of symmetry for the streams. The so-called energy output in the amplification state is also calculated

$$W_y = J\alpha_a L_y s_p/G,$$

where α_a is the average integral value of α in the resonator.

The obtained value for the energy output is a function of the resonator parameters $W = W(J, L_x, L_y)$, $W_y = W_y(J, L_x)$. If it is possible to use the parameters of the resonator for optimization, i.e., for selecting maximum energy output, then the corresponding parameters are not indicated in the definition. For example, when the energy output is optimized along the length of the resonator, we will use the definition $W(J, L_y)$, $W_y(J)$.

Such a model for the processes in the resonator does not account for the interaction of radiation with ripple inhomogeneities in the medium and describes interaction with steady-state inhomogeneities in an overly simplified manner. Taking into account such a factor reveals the characteristics of radiation, especially in the cases of small distances from the nozzle edges to the resonator. This problem requires further investigation.

In order to avoid the negative factors indicated above, one must position the resonator only in the region where complete mixing of the gases occurs and all of the parameters are equalized. Hence, for obtaining maximum energy output it is necessary to select the parameters so that the relaxation losses before the input of the resonator are minimal. According to [6], the ripple densities, which are caused by the turbulent character of mixing in a resonator positioned in the zone after the mixing of the gases, are sufficiently small, and one can neglect their effect on the optical characteristics [6].

2. The initial parameters for the basic calculation variants are given in Table 1. The index I is related to the stream of CO_2 and H_2O, and the index E pertains to the stream of N_2. For the initial cross section in stream I $c(CO_2) = 0.96$, $c(H_2O) = 0.04$ with the exception of variant 13 (see below); in stream E $c(N_2) = 1$, $u = 1878$ m/sec. The flow temperatures are $T_I = T_E = 300$ K, the half-height of stream I is 0.451 mm, and the half-height of stream E is 0.805 mm. In addition to those indicated in Table 1, variants 10 and 11 were studied, which differ from 7 and 9 in that the height of the streams is two times greater; variant 12 has a stream height which is 4 times greater than that for 9; variant 13 differs from 7 in relation to the composition of stream I: $c(CO_2) = c(N_2) = 0.48$, $c(H_2O) = 0.04$. The data for the remaining cases can be found from the information indicated below. The state parameters were selected so that one could study the effect of the initial gas dynamic parameters for the velocities, the pressures, the vibrational temperature for N_2, and the heights of the streams at the energy output.

The distance to the front of the resonator is equal to 17.5 mm unless otherwise mentioned. Different resonator parameters were used in making calculations; in Figs. 1 and 2 $L_y = 0.5$; 1; 2; 3 m (lines I-IV, respectively).

TABLE 1

Number of variant	p_I, Pa	p_E, Pa	u_I, m/sec	T_4, K	Number of variant	p_I, Pa	p_E, Pa	u_I, m/sec	T_4, K
1	656	656	939	2000	6	1 312	1312	939	2000
2	328	656	1878	2000	7	6 560	6560	939	2000
3	1312	656	465	2000	8	13 120	6560	465	2000
4	656	656	939	3000	9	6 560	6560	939	3000
5	328	328	939	2000					

The pressure is extremely small in variants 1-6, and it was not clear earlier whether one could use the "mode" approximation for describing the kinetic processes or whether it was necessary to use the level model of kinetic theory. Above all, this was of concern in regards to N_2, where "Boltsmannization" of the vibrations occurs much slower than it does in CO_2 [7]. However, a number of circumstances indicate that it is possible to apply the "mode" approximation. First, the initial distribution of the vibrational temperatures at the input to the nozzle in the case of thermal pumping is a Boltzmann distribution, which is not true for electron shock pumping [7] or for chemical reaction processes. Second, a comparison between the different calculation results and the experimental data in [8] shows that the level model can even give worse results than the "mode" model. This is because the constants from the level model are obtained indirectly after complicated processing of the experimental data, while the constants for the "mode" model are more directly related to the experimental results. It is assumed that the mixing in variants 1-6 is laminar, but for the remaining variants the mixing is turbulent (see also [2]).

It is clear from the calculations that for small resonator lengths L_x the difference between the energy outputs for various L_y and for the amplification state is small, but for large resonator lengths the difference is substantial; the optimal length L_x is greater when L_y is increased (see Fig. 1, where the results for variants 7 and 13 are given, $J = 5 \cdot 10^8$ W/m^2, ℓ is the sum of the half-heights for streams I and E, numbers I-IV are related to $W_I(J, L_x, L_y)$, $W_{II}(J, L_x, L_y)$, etc. and $\langle\alpha\rangle$ is the average amplification factor). In [9], which studies the efficiency of an HDL used with a heat exchanger, the optimal resonator length L_y is 1.3 m for $L_xL_y = 0.25$ m^2. However, for independent variations of these parameters the energy output monotonically increases with an increase in L_y.

In variant 7 the length of the generation zone is very small (see Fig. 1) because of the high CO_2 concentration. It is possible to increase the length of the generation zone by diluting the CO_2 concentration, where N_2 is used with the same temperature (300°K). Substituting half the weight of the CO_2 content by N_2 (variant 13) leads to an increase in the energy output for W_{IV}, W_{III}, W_y, in addition to an increase in the length of the generation zone and rapid equalization of the densities due to the smaller difference between the molecular weights of the streams.

Values for the energy outputs are shown in Fig. 2 for optimal lengths of the resonators and for an optimal transmission factor of the mirror. For small radiation intensities $W_y(J)$ and $W(J, L_y)$ differ very little, but with an increase in intensity the difference increases and the optimal value of the mirror's transmission factor falls. The values of W_y and $W(L_y)$ for $L_y = 3$ m are optimized with respect to the intensity and differ very little, where the difference for all the variants fluctuates in the range of 4-15%. Therefore, the values of W_y can be used to describe the upper limit of the energy output for large L_y. The insignificant differences between the energy outputs for the different variants with $L_y = 2$ and 3 m (2-5%) are insignificant, i.e., an increase in the width of the resonator to over 2 m has little effect. However, the difference between the values of W for the different variants with $L_y = 0.5$; 1; 2 m is substantial. One should note that the results for the effect of L_y on the energy output can change significantly in dependence on the absorption coefficients of the mirrors.

For the undesigned variants, the flow contains a system of shock waves and low pressure waves [10]. The equalization of all the flow parameters in the variants with turbulent mixing occurs at lengths at 45-90 calibers, where the most rapid equalization occurs in variant 8 with large velocity differences. The profiles for the dimensionless flow parameters at the input to the resonator are shown for variant 7 in Fig. 3 (the parameters in stream I are used as gauges, and the gauge for the pressure is $\rho_I u_I^2$) — the gases are still unmixed,

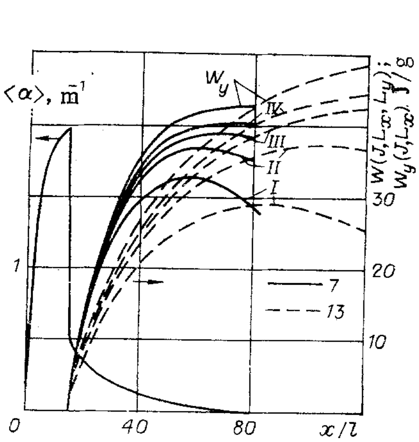

Fig. 1

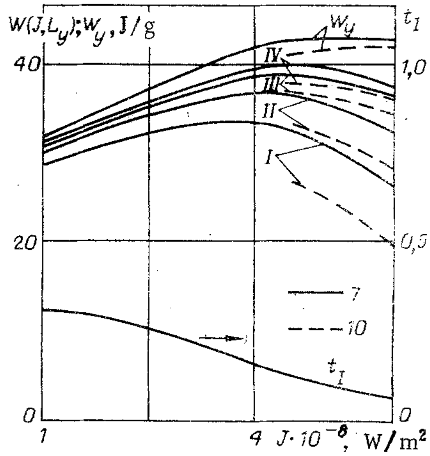

Fig. 2

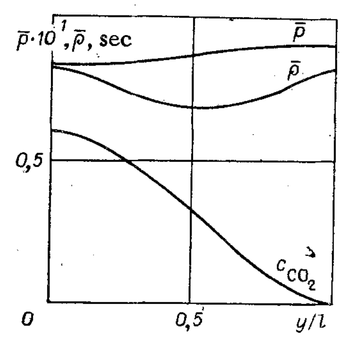

Fig. 3

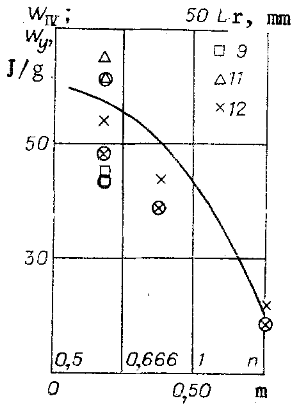

Fig. 4

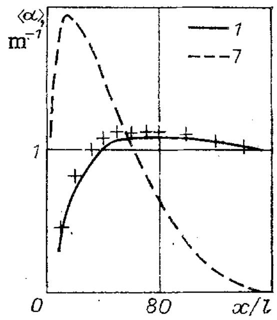

Fig. 5

and there is a "depression" in the density profile which is related to heating in the mixing layer; it is evident that even in the designed variant there are small changes in the pressure across the flow.

For understanding the role of undesigned streams and the dissipation of kinetic energy into thermal energy a number of calculations were made using variant 7 as the base variant; for the remaining variants the pressure and velocity in the stream of $CO_2 + H_2O$ was varied so that the flow rate was constant. These results are given in Fig. 4 (the solid lines), where $M = (u_E - u_I)/u_E$, $n = p_I/p_E$; they indicate that the undesigned variant where M is minimum is preferable, i.e., where the transformation of kinetic energy into thermal energy is minimum.

The effect of pressure on the energy output can be illustrated by the following variants: for a variant corresponding to all parameters, except pressure, along with variant 7 (the pressure is two times lower), W_y = 46.5 J/g; variant 7 W_y = 42 J/g and a variant in which the pressure is two times higher than in 7, W_y = 34.8 J/q (the values of the energy output are optimized with respect to L_x and J).

The given calculations (Fig. 4, L_r is the distance up to the resonator, the circled data points represent W_{IV}, and the data points without circles are for W_y) show that the energy output is a function of the distance from the nozzle edge to the resonator and is greater with an increase in the intensity of the relaxation processes in the system, which, therefore, leads to a larger loss of vibrational energy before the input of the resonator. For T_4 = 3000 K, M = 0.5 one can obtain a gain only by positioning the resonator close to the edge of the nozzle. One should note that it is possible to increase T_4 without changing the other parameters at the edge of the nozzle if the HDL uses nonequilibrium electric-arc heating of N_2 [11].

TABLE 2

Number of variant	$W_y(J)$, J/g	Number of variant	$W_y(J)$, J/g
1	47	5	42
2	61	6	48
3	27	7	18
4	90	9	15

For obtaining maximum energy output it is necessary to position the resonator immediately after the zone where initial mixing occurs, i.e., the nitrogen diffuses up to the plane of symmetry for the streams of CO_2 (hence, there is no absorption in the gas); in addition, the mixing must ensure a sufficient intake of vibrational energy (which was the case in the variants we used). The end of the initial section of mixing in the designed variants was situated at a length of 2.3 calibers, which for variants of different stream widths is 3-11.5 mm, and the average amplification factor is greater than the threshold over a length of several millimeters (see Fig. 1). In addition, one must consider that it is complicated to reduce the distance between the resonator and the edge of the nozzles below some constructive limit.

An increase in the dimensions of the mixed streams for T_4 = 2000 K (see Fig. 2) leads to a drop in the energy output, which is especially noticeable for small L_y. However, for T_4 = 3000 K (see Fig. 4) there is an optimal stream height $\approx$2.5 mm, which is caused by intense relaxation, and for a reduction in the dimension of the streams the mixing accelerates, where the fraction of the vibrational energy lost up to the input of the resonator increases.

Therefore, when the relaxation is intense and the distance from the edge of the nozzles to the resonator is fixed, there is an optimum dimension for the nozzles. One should also note there is a lower limit to the dimensions of the streams when one takes into account the boundary layers in the nozzles. The generation zone is extended when the dimensions of the streams are increased (the zone is increased by about 1.5 times in variants 11 and 12) due to the increase in the rate of mixing.

The calculation results for the average amplification factors are given in Fig. 5. The crosses represent data using variant 1 in the assumption that the mixing is turbulent (hence, the mixing accelerates, but the curve for the amplification factor is close to the curve calculated with the assumption of laminar mixing).

Calculations were conducted in the amplification state for a number of variants. The results of these calculations for an intense field equal to $5 \cdot 10^7$ W/m^2 is shown in Table 2. The energy output is significant for low pressures; for high pressures the intensity is taken to be far from the optimum because of the small energy output. One should note that substantially differing behavior is observed between the results obtained with and without resonator optimization. As is evident in Table 2, the parameter M has a strong effect on the energy output. From a comparison of the results for variants 1, 5, and 6 it is clear that for the used value of the intensity the optimum pressure is 1300 Pa, but the optimum value of the intensity increases with an increase in the pressure. The lengths of generation for low pressures significantly increase up to 300-350 calibers.

In a HDL where the components have been previously mixed and heated in a heat exchanger at $T_0 \leq$ 2000 K, calculations show that the energy output is 15-20 J/g [9]. The results of this study show that it is possible to obtain 40-60 J/g for a HDL with selective thermal excitation at the vibrational temperature of N_2, equal to 2000°K, for nearly optimal parameters. These results were obtained on the basis of a realistic model of mixing. They indicate the effectiveness of the assumed numerical model for investigating the mixing of gases taking into account vibrational nonequilibrium.

LITERATURE CITED

1. M. G. Anan'evskii, E. T. Antropov, et al., "The application of a hydrodynamic laser with a high-temperature, regenerative heat exchanging heater in metallurgy," Teplofiz. Vys. Temp., No. 2 (1981).
2. G. N. Volchkova, A. V. Lavrov, and S. S. Kharchenko, "Application of the simplified Navier–Stokes equations for making calculations of laminar mixing of vibrationally relaxing gases," Teplofiz. Vys. Temp., No. 6 (1981).

3. N. I. Akatnov, "Dual-scale semiempirical theory of turbulent boundary layers and streams," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1982).
4. S. A. Losev, Hydrodynamic Lasers [in Russian], Nauka, Moscow (1977).
5. S. V. Kulikov, "Comparison of the calculations for the power of a CO_2 HDL made under different steady-state generation conditions," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1980).
6. P. Kassadi, J. Newton, and P. Rouz, "A new type of mixing hydrodynamic laser," Raket. Tekh. Kosmon., 16, No. 4 (1978).
7. V. A. Pivovar, "Limited application for an approximation of the vibrational temperature for nitrogen in a short pulse duration CO_2–N_2–He laser," Zh. Tekh. Fiz., 47, No. 2 (1977).
8. B. V. Egorov and V. N. Komarov, "A comparative analysis of the kinetic models for vibrational relaxation in mixtures of gases containing CO_2," ChMMSS, 11, No. 3 (1980).
9. E. T. Antropov, V. T. Karpukhin, and Yu. B. Konev, "A theoretical investigation of the characteristics of high temperature hydrodynamic lasers," Preprint Institute of HIgh Temperatures, Akad. Nauk SSSR, Nos. 5-37 (1979).
10. T. A. Bungova, A. V. Lavrov, T. A. Spas, and S. S. Kharchenko, "Nonequilibrium physical and chemical processes in a system of two-dimensional turbulent, undesigned streams," in: Stream Flows of Liquids and Gases [in Russian], Novopolotsk (1982).
11. B. V. Abakumov, Yu. V. Kurochkin, et al., "A hydrodynamic laser with thermally nonequilibrium electric-arc heating," Kvantovaya Elektron., No. 9 (1979).

CALCULATION OF NON-STEADY-STATE FLOWS OF RELAXED GASES IN CHANNELS

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Maintaining control at supersonic velocities in the atmosphere requires experimental investigations of flow around aircraft for a wide range of Mach and Reynolds numbers. One technique for modeling such flow involves the use of a high-enthalpy wind tunnel. Examples of such set-ups include pulse-driven tunnels with gas heated by a discharge in a confined volume [1] and tunnels with electric-arc heaters [2]. In the first case all of the working processes are non-steady-state, and in the second case one begins with non-steady-state output where the established flow is in the ripple state. It is natural to assume that in the given cases the nonlinear interaction of wave structures has a significant effect on the formation of the flow of a real gas. Such an interaction arises when the diaphragm in a tunnel breaks down and when the energy input is periodic. Of particular importance are the problems in [3], which are related to pulsed heating of the CO_2 flow by an electrical discharge with a small duration. It was shown in [4] that the modified technique of S. K. Godunov is an effective way for studying similar flows. In this study Godunov's technique relating to flows with external energy input [4] is applied for calculations of flows with vibrational relaxation. This technique is shown to be useful for numerical modeling of steady-state flows in nozzles, non-steady-state flows in pulse-driven wind tunnels, and for flows with periodic energy input into the subsonic zone of a channel, which reproduces the flowing part of a coaxial heater and a supersonic nozzle of a typical high-enthalpy wind tunnel. Questions regarding similarity and the modeling of non-steady-state flows with vibrational relaxation are considered.

1. Nonequilibrium energy exchange between vibrational and active degrees of freedom for molecules (the V–T process) reflects the basic behavior of relaxation phenomena and allows one to easily model the effect of relaxation on the flow of a high temperature gas. On the other hand, such a calculation technique can be used for solving a variety of problems related to nonequilibrium flow of carbon dioxide and carbon dioxide mixtures [5].

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